Vortices in a nonminimal
Maxwell-Chern-Simons O(3) Sigma Model

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Abstract

In this work we consider an Abelian O(3) sigma model coupled nonminimally with
a gauge field governed by Maxwell and Chern-Simons terms. Bogomol'nyi equations
are constructed for a *specific form* of the potential and generic nonminimal coupling
constant. Furthermore, topological and nontopological self-dual soliton solutions are
obtained for a critical value of the nonminimal coupling constant. Some particular
static vortex solutions (topological and nontopological) satisfying the Bogomol'nyi
bound are numerically solved and presented.

1   INTRODUCTION

The gauging of a non-linear sigma model, giving rise to a coupling between
scalar (coordinates of the target space) and gauge fields has attracted interest
from different areas. In particular, soliton solutions of the gauged O(3) Chern-
Simons model may be relevant in planar condensed matter systems [1–3].
Recently, self-dual solutions were obtained in a (2+1) dimensions gauged O(3)
sigma model [4]. In the context of Skyrme models the requirement of gauge
symmetry was used to break the scale invariance of the O(3) sigma model with
gauge field dynamics governed by a Maxwell term, where the gauge group U(1)
is a subgroup of O(3), giving the so called baby Skyrme model [5,6] (aided by a
potential term). Other works, in which the gauge field dynamics is governed by
a Chern-Simons term, were studied in a series of papers [7–9]. These systems

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becomes self-dual for a specific choice of the Higgs potential and topological as well as nontopological solutions are present [10].

Stern [11] was the first one to suggest a nonminimal term in the context of the Maxwell-Chern-Simons electrodynamics intending to mimic an anyonic behavior without a pure Chern-Simons limit. This term could be interpreted as a generalization of the Pauli coupling, *i. e.*, an anomalous magnetic moment. It is a specific feature of (2+1) dimensions that the Pauli coupling exists not only for spinning particles but for scalar ones too [12–15]. As a fundamental result, Stern showed that, for a particular value of the nonminimal coupling constant, the field equations of his model have the same form of the field equations of a *pure* Chern-Simons theory minimally coupled.

In this work we consider an *Abelian* gauged O(3) sigma model with a non-minimal coupling, where we have included Maxwell and Chern-Simons terms. Self-dual soliton solutions are obtained for this model at a critical value for the nonminimal coupling constant. Our soliton solutions are charged, but the magnetic flux is topologically quantized only for topological solitons (where stability is guaranteed by topological arguments). It is important to emphasize that the Bogomol'nyi bound [16] will be achieved with the introduction of a neutral scalar field. Previously, a *non-Abelian* gauged sigma model with anomalous magnetic moment was discussed [17]. However that work did not considered the Higgs potential.

Finally, we give the asymptotic behaviour of the topological and the nontopological solitons and present the numerically calculated soliton profiles for *both* cases.

2   THE NONMINIMAL GAUGED O(3) SIGMA MODEL

In the O(3) sigma model the scalar field $\boldsymbol{\phi}$ is a map from the (2+1)-dimensional Minkowski space to the two-sphere of unit radius denoted by S^2. In other words, $\boldsymbol{\phi}$ is a three component vector satisfying the constraint $\boldsymbol{\phi} \cdot \boldsymbol{\phi} = \phi_1^2 + \phi_2^2 + \phi_3^2 = 1$. Now we look for a Lagrangian invariant under global iso-rotations of the field $\boldsymbol{\phi}$ about a fixed axis $\mathbf{n} \in S^2$. In order to gauge this symmetry, we choose $\mathbf{n} = (0, 0, 1)$ and select the local U(1) subgroup of the O(3) rotational group.

Therefore, the nonminimal gauged O(3) sigma model is defined by the following Lagrangian density

$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{\kappa}{4}\varepsilon^{\mu\nu\alpha}A_\mu F_{\nu\alpha} + \frac{1}{2}\partial_\mu M\partial^\mu M - g\partial_\mu M\partial^\mu(\mathbf{n}\cdot\boldsymbol{\phi})$$
$$+\frac{1}{2}\nabla_\mu\boldsymbol{\phi}\cdot\nabla^\mu\boldsymbol{\phi} - U(M, \mathbf{n}\cdot\boldsymbol{\phi}), \tag{1}$$

where the second term is the called Chern-Simons term and the minimal gauge covariant derivative $D_\mu\boldsymbol{\phi} \equiv \partial_\mu\boldsymbol{\phi} + [eA_\mu]\mathbf{n}\times\boldsymbol{\phi}$ was changed by

$$\nabla_\mu\boldsymbol{\phi} = \partial_\mu\boldsymbol{\phi} + \left[eA_\mu + \frac{g}{2}\varepsilon_{\mu\nu\alpha}F^{\nu\alpha}\right]\mathbf{n}\times\boldsymbol{\phi} \qquad . \tag{2}$$

The real scalar field M is introduced for convenience, as explained later. This Lagrangian is clearly gauge invariant since the potential depends only on ϕ_3, which is gauge invariant.

Notice that our metric is $\eta_{\mu\nu} = diag(+1, -1, -1)$ with $\varepsilon^{012} = 1$; $\mu, \nu = 0, 1, 2$ and $i, j = 1, 2$.

The equations of motion for the gauge field follows from (1) as

$$e\mathbf{n}\cdot\boldsymbol{\phi}\times\nabla^\mu\boldsymbol{\phi} + \kappa F^\mu - \varepsilon^{\mu\nu\alpha}\partial_\alpha\left(g\mathbf{n}\cdot\boldsymbol{\phi}\times\nabla_\nu\boldsymbol{\phi} - F_\nu\right) = 0 \tag{3}$$

where $F^\mu \equiv \frac{1}{2}\varepsilon^{\mu\nu\alpha}F_{\nu\alpha}$ is the dual to the field strength and the matter current is given by

$$J^\mu = -e\mathbf{n}\cdot\boldsymbol{\phi}\times\nabla^\mu\boldsymbol{\phi} \qquad . \tag{4}$$

So, equation (3) can be rewritten as

$$\partial_\mu\left[\varepsilon^{\mu\alpha\nu}\left(\frac{g}{e}J_\nu + F_\nu\right)\right] = J^\alpha - \kappa F^\alpha \qquad . \tag{5}$$

The solutions of the first order differential equation

$$F^\alpha = \frac{1}{\kappa}J^\alpha \tag{6}$$

are also solutions of (5) provided that

$$g = -\frac{e}{\kappa} \tag{7}$$

holds.

This result is completely analogous to that presented by Stern [11] and later by Torres [14] in the context of Maxwell-Chern-Simons electrodynamics.

It is worth mentioning that the $\alpha = 0$ component of the equation (6) is the corresponding *Gauss law,* namely

$$J_0 + \kappa B + \partial_i E_i + \frac{g}{e}\varepsilon_{ij}\partial_i J_j = 0 \tag{8}$$

where B and E_i are the magnetic and electric fields respectively.

Integration of (8) in the whole space leads to

$$\Phi \equiv \int d^2 x B = -\frac{q}{\kappa} \ . \tag{9}$$

Therefore, charged vortices with charge q are also tubes of magnetic flux.

Now we construct the energy functional. In this direction, a matter field equation of motion is required and it is possible to express it in terms of currents. So, let us consider the following equation of motion for the scalar field

$$\nabla_\mu \nabla^\mu \boldsymbol{\phi} = -\frac{\partial V}{\partial \boldsymbol{\phi}} \ . \tag{10}$$

We define a new matter current (which is a vector in the target space) as

$$\mathbf{j}^\mu = -e \boldsymbol{\phi} \times \nabla^\mu \boldsymbol{\phi} \ . \tag{11}$$

Thus the equation (4) can be rewritten in the following form

$$J^\mu = \mathbf{n} \cdot \mathbf{j}^\mu \qquad . \tag{12}$$

Now the equation of motion (10) turns to

$$\nabla^\mu J_\mu = e \boldsymbol{\phi} \times \frac{\partial V}{\partial \boldsymbol{\phi}} \qquad , \tag{13}$$

where we have used (11) and (12). For the sake of completeness it is interesting to express the current J^μ in terms of a current without explicit magnetic moment contribution, denoted by $k^\mu \equiv -e\mathbf{n} \cdot \boldsymbol{\phi} \times D^\mu \boldsymbol{\phi}$. So

$$k^\mu = J^\mu \left[1 + (g\mathbf{n} \times \boldsymbol{\phi})^2 \right] \ . \tag{14}$$

Later the expression above will be useful to write the energy functional properly.

In addition, a gauge invariant topological current J_{top}^μ can be defined as

$$J_{top}^\mu = \frac{1}{8\pi} \varepsilon^{\mu\nu\alpha} \boldsymbol{\phi} \cdot \left[D_\nu \boldsymbol{\phi} \times D_\alpha \boldsymbol{\phi} - \frac{e}{2} F_{\nu\alpha} \left(1 - \mathbf{n} \cdot \boldsymbol{\phi} \right) \boldsymbol{\phi} \right] \ , \tag{15}$$

which leads to a topological charge in the form

$$Q_{top} = \int d^2 x J_{top}^0 = \frac{1}{4\pi} \int d^2 x \left[\boldsymbol{\phi} \cdot D_1 \boldsymbol{\phi} \times D_2 \boldsymbol{\phi} + \frac{eB}{2} \left(1 - \mathbf{n} \cdot \boldsymbol{\phi} \right) \right] \ . \tag{16}$$

3 SELF-DUAL EQUATIONS

Usually, the energy functional is obtained by the integral over the T_{00} component of the energy momentum-tensor, which can be obtained by coupling the fields to gravity and then varying the action with respect to the metric. Taking into account that the Chern-Simons term does not contribute because of its metric independence, we have

$$
T_{\mu\nu} = G(g, \boldsymbol{\phi}) F_\mu{}^\alpha F_{\alpha\nu} + D_\mu \boldsymbol{\phi} \cdot D_\nu \boldsymbol{\phi} + \partial_\mu M \partial_\nu M
$$
$$
- \frac{1}{2} \left[\partial_\mu M \partial_\nu (\mathbf{n} \cdot \boldsymbol{\phi}) + \partial_\nu M \partial_\mu (\mathbf{n} \cdot \boldsymbol{\phi}) \right] - \eta_{\mu\nu} L \ , \tag{17}
$$

where $G(g, \boldsymbol{\phi}) = 1 - g^2 (\mathbf{n} \times \boldsymbol{\phi})^2$.

So, the T_{00} component of the energy-momentum tensor can be written as

$$
T_{00} = \frac{1}{2} G \left(B^2 + E^2 \right) + \frac{1}{2} \partial_0 M \partial_0 M + \frac{1}{2} \partial_i M \partial_i M - g \partial_0 M \partial_0 (\mathbf{n} \cdot \boldsymbol{\phi})
$$
$$
- g \partial_i M \partial_i (\mathbf{n} \cdot \boldsymbol{\phi}) + \frac{1}{2} D_0 \boldsymbol{\phi} \cdot D_0 \boldsymbol{\phi} + \frac{1}{2} D_i \boldsymbol{\phi} \cdot D_i \boldsymbol{\phi} + U(M, \mathbf{n} \cdot \boldsymbol{\phi}) \ . \tag{18}
$$

If we use the equations (2) and (14) we obtain the following expression

$$
\frac{1}{2} D_i \boldsymbol{\phi} \cdot D_i \boldsymbol{\phi} = -\frac{1}{2} g^2 (\mathbf{n} \times \boldsymbol{\phi})^2 E^2 + \frac{g}{e} F_i k_i \mp g E_i \partial_i (\mathbf{n} \cdot \boldsymbol{\phi})
$$
$$
+ \frac{1}{2} (\nabla_1 \boldsymbol{\phi} \pm \boldsymbol{\phi} \times \nabla_2 \boldsymbol{\phi})^2 \pm \boldsymbol{\phi} \cdot D_1 \boldsymbol{\phi} \times D_2 \boldsymbol{\phi} \ .
$$

Then the total energy $\mathcal{E}$ can be expressed as

$$
\mathcal{E} = \int d^2 x \left\{ \frac{1}{2} G \left[B \mp \frac{e}{G} \left((1 - \mathbf{n} \cdot \boldsymbol{\phi}) + \frac{\kappa}{e} M + g (\mathbf{n} \times \boldsymbol{\phi})^2 M \right) \right]^2 \right.
$$
$$
\pm e B \left[(1 - \mathbf{n} \cdot \boldsymbol{\phi}) + \frac{\kappa}{e} M + g (\mathbf{n} \times \boldsymbol{\phi})^2 M \right] + \frac{1}{2} (E_i \pm \partial_i M)^2
$$
$$
\mp E_i \partial_i M - \frac{1}{2} g^2 (\mathbf{n} \times \boldsymbol{\phi})^2 E^2 + \frac{1}{2} \partial_0 M \partial_0 M - \frac{1}{2} g \partial_0 M \partial_0 (\mathbf{n} \cdot \boldsymbol{\phi})
$$
$$
- g \partial_i M \partial_i (\mathbf{n} \cdot \boldsymbol{\phi}) + \frac{1}{2} |D_0 \phi \pm e M (\mathbf{n} \times \boldsymbol{\phi})|^2
$$
$$
\pm M \left(J_0 - e g B (\mathbf{n} \times \boldsymbol{\phi})^2 \right) - \frac{1}{2} g^2 (\mathbf{n} \times \boldsymbol{\phi})^2 E^2 + \frac{g}{e} F_i k_i
$$
$$
\left. \mp g E_i \partial_i (\mathbf{n} \cdot \boldsymbol{\phi}) + \frac{1}{2} (\nabla_1 \boldsymbol{\phi} \pm \boldsymbol{\phi} \times \nabla_2 \boldsymbol{\phi}) \pm \boldsymbol{\phi} \cdot D_1 \boldsymbol{\phi} \times D_2 \boldsymbol{\phi} \right\} \ . \tag{19}
$$

In order to make the system self-dual, we have chosen the potential as

$$U = \frac{e^2}{2G} \left[(1 - \mathbf{n} \cdot \boldsymbol{\phi}) + \frac{\kappa}{e} M + g \left(\mathbf{n} \times \boldsymbol{\phi} \right)^2 M \right]^2 + \frac{1}{2} e^2 M^2 \left(\mathbf{n} \times \boldsymbol{\phi} \right)^2 \qquad (20)$$

and we required that the following conditions must be satisfied

$$B \mp \frac{e}{G} \left((1 - \mathbf{n} \cdot \boldsymbol{\phi}) + \frac{\kappa}{e} M + g \left(\mathbf{n} \times \boldsymbol{\phi} \right)^2 M \right) = 0 \qquad (21)$$

$$E_i \pm \partial_i M = 0 \qquad (22)$$

$$D_0 \boldsymbol{\phi} \pm eM \left(\mathbf{n} \times \boldsymbol{\phi} \right) = 0 \qquad (23)$$

$$\nabla_1 \boldsymbol{\phi} \pm \boldsymbol{\phi} \times \nabla_2 \boldsymbol{\phi} = 0 . \qquad (24)$$

Then we have a new form for the energy functional, namely

$$\mathcal{E} = \int d^2x [E_i \left(\frac{g}{e} \varepsilon_{ij} k_j - g^2 \left(\mathbf{n} \times \boldsymbol{\phi} \right)^2 E_i \right) + \frac{1}{2} \partial_0 M \partial_0 M$$

$$- \frac{1}{2} g \partial_0 M \partial_0 (\mathbf{n} \cdot \boldsymbol{\phi}) \pm M \left[J_0 + \kappa B + \partial_i E_i \right] \mp \partial_i \left(E_i M \right)$$

$$\mp g \left(E_i \pm \partial_i M \right) \partial_i (\mathbf{n} \cdot \boldsymbol{\phi})] + 4\pi \int d^2x J_{top}^0 \qquad (25)$$

The last term in the expression above is just $4\pi Q_{top}$, where Q_{top} is the topological charge defined in eq.(16).

Using the equation (22), integrating by parts and considering that the surface terms goes to zero at infinity, we have

$$\mathcal{E} = 4\pi \left| Q_{top} \right| \pm \int d^2x \left[M \left(J_0 + \kappa B + \partial_i E_i + \frac{g}{e} \varepsilon_{ij} \partial_i J_j \right) + \frac{1}{2} \partial_0 M \partial_0 M \right.$$

$$\left. - \frac{1}{2} g \partial_0 M \partial_0 (\mathbf{n} \cdot \boldsymbol{\phi}) \right]$$

Now taking into account the Gauss law (8) and considering static solutions, clearly the energy functional is bounded by below as

$$\mathcal{E} \geq 4\pi \left| Q_{top} \right| \qquad (26)$$

and this bound is saturated when the Bogomol'nyi equations (21-24) are satisfied.

It is worth mentioning the role played by the neutral scalar field M. In fact, it is essential to obtain a self-dual model, as was first pointed by Lee *et al.* [18] in the context of the Maxwell-Chern-Simons-Higgs model. Still in this context, but with nonminimal coupling, similar result was presented in refs. [19].

The requirement for finite energy solutions restrict our boundary conditions. From the potential (20) we see that the boundary condition is

$$\lim_{r \to \infty} \boldsymbol{\phi} = \pm \mathbf{n} \ . \tag{27}$$

4   STATIC VORTEX SOLUTIONS

Next, we solve the problem posed by the self-dual equations (21-24). As we have seen, in the limit $g = g_c = -\frac{e}{\kappa}$, the equations of motion for the gauge field become first order equations. Henceforth we restrict ourselves to this limit. From the $\alpha = 0$ component of the equation (6) and using (4), we obtain the following expression for the magnetic field B

$$B = \pm \frac{e g_c M (\mathbf{n} \times \boldsymbol{\phi})^2}{\left[1 - g_c^2 (\mathbf{n} \times \boldsymbol{\phi})^2\right]} \ . \tag{28}$$

Note that we have used that $A_0 = \mp M$, which can be see from (22) for static solutions.

On the other hand, the Bogomol'nyi equations (21-24) provide

$$B = \mp \frac{e}{g_c} M \pm \frac{e(1 - \mathbf{n} \cdot \boldsymbol{\phi})}{1 - g_c^2 (\mathbf{n} \times \boldsymbol{\phi})^2} \ . \tag{29}$$

Therefore, in this limit, the scalar field M may be written in terms of just the Higgs field, namely

$$M = g_c \left(1 - \mathbf{n} \cdot \boldsymbol{\phi}\right)$$

and consequently

$$B = \pm e g_c \frac{(\mathbf{n} \times \boldsymbol{\phi})^2}{1 - g_c^2 (\mathbf{n} \times \boldsymbol{\phi})^2} \left(1 - \mathbf{n} \cdot \boldsymbol{\phi}\right) \ . \tag{30}$$

Now, the potential (20) can be rewritten as

$$U = \frac{e^2 g_c^2}{2} \frac{(1 - \mathbf{n} \cdot \boldsymbol{\phi})^2}{[1 - g_c^2 (\mathbf{n} \times \boldsymbol{\phi})^2]} \left[1 - (\mathbf{n} \cdot \boldsymbol{\phi})^2\right] \ . \tag{31}$$

Note that in the limit $g_c \to 0$, this potential becomes the one considered in Ref. [7].

We search solutions which are invariant under simultaneous rotations and reflections in space-time and target space. So, the spherically symmetric fields

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are obtained by making use of the *ansatz* [5]

$$\boldsymbol{\phi}(x) = (\sin f(r) \cos N\theta, \sin f(r) \sin N\theta, \cos f(r)) \tag{32}$$

where (r, θ) are polar coordinates in the **x**-plane, N is a non-zero integer and also defines the degree of a topological soliton (*vorticity*). To the gauge field given by $\mathbf{A} = N\frac{a(r)}{er}\widehat{\theta}$, the electric field vanishes, and the magnetic field is such that $B = N\frac{a'}{er}$, where primes denote differentiation with respect to r. Substitution of this *ansatz* into equations (23), (24) and (30) gives

$$f'(r) = \pm\frac{N}{r}\frac{\sin f(r)}{1 + g_c^2 \sin^2 f(r)} \left[1 + a(r)\right] , \tag{33}$$

$$a'(r) = \pm\frac{r}{N}\frac{g_c^2 \sin^2 f(r)}{1 - g_c^2 \sin^2 f(r)} \left(1 - \cos f(r)\right) , \tag{34}$$

$$\frac{1}{e}B = \pm g_c^2 \frac{\sin^2 f(r)}{1 - g_c^2 \sin^2 f(r)} \left(1 - \cos f(r)\right) . \tag{35}$$

In order to obtain solutions of the above equations it is necessary to consider the asymptotic limit of the functions. The equations (33) and (34) can be decoupled and we will have a second order equation for $f(r)$ given by

$$f''(r) = -\frac{1}{r}f'(r) + \frac{[f'(r)]^2}{\tan f(r)} - \frac{g_c^2 \sin 2f(r)}{\left[1 + g_c^2 \sin^2 f(r)\right]^2} [f'(r)]^2$$
$$+ \frac{g_c^2 \sin^3 f(r)}{1 - g_c^4 \sin^4 f(r)} \left(1 - \cos f(r)\right) . \tag{36}$$

Now we will find the solutions of the equations (33) and (34). First we consider the field variables near the origin. To ensure that the field is nonsingular at origin we impose

$$f(0) = n\pi, \ n \in \mathbb{N} . \tag{37}$$

Since $\mathbf{A} = N\frac{a(r)}{er}\widehat{\theta}$, clearly regular solutions at origin require $a(0) = 0$. On the other hand, solutions for $f(r)$ are symmetric about $f(r) = 2\pi$. Therefore we can choose $f(0) = 0$ and $f(0) = \pi$. If we consider the latter condition it is interesting rewrite $f(r)$ as $f(r) = \pi + h(r)$. Considering also the lower sign and *positive N*, the expression (33) for $h(r) \ll 1$ admits solution like

$$h(r) = C_0 r^N . \tag{38}$$

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Consequently

$$a(r) = -\frac{1}{2N(2N+1)}g_c^2 r^{4N+2} \ . \tag{39}$$

If we consider the former boundary condition $f(0) = 0$ and *negative N*, near the origin, solutions of the eq. (33) are

$$f(r) = \overline{C}_0 r^{-N} \ . \tag{40}$$

Further, in this case, a solution for $a(r)$ may be written as

$$a(r) = -\frac{\overline{C}_0}{4N(1-2N)}g_c^2 r^{-4N+2} \ . \tag{41}$$

On the other hand, at infinity, two distinct asymptotic behaviours for the solutions of (36) are possible. When $f(\infty) = \pi$ and N is *positive*, again $f(r)$ can be written as $f(r) = \pi + h(r)$. In addition, an asymptotic behaviour for $a(r \to \infty)$ compatible with finite energy and eq. (34) requires $a(r \to \infty) \equiv -\alpha_1$. Finite solutions of the eq. (36) are of the form

$$h(r) = C_\infty r^{N(1-\alpha_1)} \ , \ \alpha_1 > 1 \tag{42}$$

and correspondingly

$$a(r) = -\frac{C_\infty^2}{2N^2(1-\alpha_1)+2N}r^{2N(1-\alpha_1)+2} - \alpha_1 \ . \tag{43}$$

Choosing the other boundary condition at infinity, namely, $f(\infty) = 0$, $a(r \to \infty) \equiv \alpha_2$ and considering N as a *negative* number, we have

$$h(r) = \overline{C}_\infty r^{N(1+\alpha_2)} \ , \ \alpha_2 > -1 \tag{44}$$

and

$$a(r) = -\frac{\overline{C}_\infty^4 g_c^2}{2N}r^{4N(1+\alpha_2)+2} + \alpha_2 \ . \tag{45}$$

Note that, in the above expressions, $\alpha_{1,2}$ are constants which depend on our choice of topological or nontopological boundary conditions and are to be considered just numerically determined.

For the ansatz (32) the topological charge can be expressed as [7]

$$Q_{top} = \frac{N}{2}\left[cosf(0) - cosf(\infty)\right] \mp \frac{\alpha_{1,2}}{2}\left[1 - cosf(\infty)\right] \ . \tag{46}$$

From the above equation one finds that solutions under conditions $f(0) = \pi$ and $f(\infty) = \pi$, have a non-integer topological charge which caracterize a

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nontopological soliton [10]. On the other hand, boundary conditions $f(0) = 0$ and $f(\infty) = \pi$ lead to solutions with $Q_{top} = N$ and so are called topological solitons. It is worth adding that topological solitons have quantized energy $\mathcal{E} = 4\pi |N|$ and not quantized magnetic flux $\Phi = \frac{2\pi}{e} N \alpha_2$. In the case of nontopological solitons, neither the energy $\mathcal{E} = 4\pi \alpha_1$ nor the magnetic flux $\Phi = \frac{2\pi}{e} N \alpha_1$ are quantized.

5 NUMERICAL RESULTS

The set of equations (33) and (34) was solved numerically for the boundary conditions discussed above. First, we look for topological solutions so we use the boundary conditions $f(0) = 0$ and $f(\infty) = \pi$ with $N = -1$. Nontopological solitons are obtained for $f(0) = \pi$ and $f(\infty) = \pi$ with $N = 1$. For constructing numerical solutions we have used a shooting method. The asymptotic behaviours (38), (39), (42) and (43) were used for nontopological solutions while expressions (40), (41), (44) and (45) were used for topological one. With regard to the latter case, the profiles for the functions $f(r)$ and $a(r)$ for four different values of the critical nonminimal coupling constant g_c are given in Figs. 1-2 respectively, whereas Figs. 4-5 show the functions $f(r)$ and $a(r)$ for the nontopological case. The magnitude of the magnetic field B as a function of r for $N = -1$ and $N = 1$ is also presented in Figs. 3 and 6 respectively. Here it is interesting to comment that for nontopological solitons (Fig. 6), the magnetic field has doubly degenerate maxima, as first pointed out by Ghosh and Ghosh [10]. There degenerate and nondegenerate maxima depend on the value of $a(\infty)$, while here this behaviour depends on the values of g_c, wherefore on the values of the relation $\frac{e}{\kappa}$.

6 CONCLUSIONS

In this paper we have considered a gauged O(3) sigma model with both Maxwell and Chern-Simons terms. Besides the minimal coupling, an extra term which couples the complex scalar field directly to the gauge field strength, was introduced. As in the context of Maxwell-Chern-Simons-Higgs systems the presence of self-dual solutions is guaranteed only by introduction of a neutral scalar field. Bogomol'nyi equations were constructed for a *specific form* of the potential and generic nonminimal coupling constant. However, in contrast with several nonminimal models existing in the literature, at a critical value for the nonminimal coupling parameter, we have obtained topological as well as non-topological soliton solutions (see however [19], where a dimensional reduction is used to construct a $N = 2 - D = 3$ Maxwell-Chern-Simons-Higgs nonmini-

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mal model). Numerical solutions for *both* topological and nontopological cases are obtained and presented.

It is known that the critical condition $g_c = -\frac{e}{\kappa}$ turns a Maxwell-Chern-Simons system in a pure Chern-Simons one [11,14]. So it is not surprising that for O(3) sigma models our results are similar to the numerical solutions of Ghosh and Ghosh [10] for an O(3) sigma model where the gauge field dynamics is solely governed by a Chern-Simons term. Indeed, for instance, the unusual behaviour of the magnetic field for nontopological solitons with doubly degenerate maxima are present in our model for specific values of g_c. To choose a value for g_c means to choose a value for the coefficient κ of the CS term, so κ can be used to define a class of solutions with degenerate or non degenerate maxima of the magnetic field.

It is worthwhile to mention that, notwithstanding the resemblance commented above, the Maxwell-Chern-Simons theories even in the large distance limit are richer than the pure Chern-Simons model [20]. On the other hand the moduli space dynamics of the solutions can be quite different due to the contribution of the Maxwell term in the Lagrangian [21].

Recalling the baby Skyrme model, the above discussed aspects of our model may be used in an analysis of a relation between Maxwell, Hopf and Skyrme terms in order to break the scale invariance of the sigma model.

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FIGURE CAPTIONS

Figure 1: A plot of $f(r)$ as a function of r for N=-1 *topological* soliton solutions for different values of the critical coupling parameter g with $g = -0.8$ (dashed-dotted line), $g = -0.6$ (long-dashed line), $g = -0.4$ (dashed line), $g = -0.2$ (solid line).

Figure 2: A plot of $a(r)$ as a function of r for N=-1 *topological* soliton solutions for different values of the critical coupling parameter g and the parameter $a(\infty) \equiv \alpha_2$ with $g = -0.8$ and $\alpha_2 = 6.3590$ (dashed-dotted line), $g = -0.6$ and $\alpha_2 = 3.7824$ (long-dashed line), $g = -0.4$ and $\alpha_2 = 2.2078$ (dashed line), $g = -0.2$ and $\alpha_2 = 1.0641$ (solid line).

Figure 3: The magnitude of the magnetic field B as a function of r for N=-1 *topological* soliton solutions for different values of the critical coupling parameter g with $g = -0.8$ (dashed-dotted line), $g = -0.6$ (long-dashed line), $g = -0.4$ (dashed line), $g = -0.2$ (solid line).

Figure 4: A plot of $f(r)$ as a function of r for N=1 *nontopological* soliton solutions for different values of the critical coupling parameter g with $g = -0.025$ (dashed-dotted line), $g = -0.02$ (long-dashed line), $g = -0.015$ (dashed line), $g = -0.01$ (solid line).

Figure 5: A plot of $a(r)$ as a function of r for N=1 *nontopological* soliton solutions for different values of the critical coupling parameter g and the parameter $a(\infty) \equiv -\alpha_1$ with $g = -0.025$ and $\alpha_1 = 4.8459$ (dashed-dotted line), $g = -0.02$ and $\alpha_1 = 5.1790$ (long-dashed line), $g = -0.015$ and $\alpha_1 = 5.9420$ (dashed line), $g = -0.01$ and $\alpha_1 = 9.3737$ (solid line).

Figure 6: The magnitude of the magnetic field B as a function of r for N=1 *nontopological* soliton solutions for different values of the critical coupling parameter g with $g = -0.025$ (dashed-dotted line), $g = -0.02$ (long-dashed line), $g = -0.015$ (dashed line), $g = -0.01$ (solid line).